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NORMAL PETRI NETS

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Abstract. A class of Petri nets, called normal Petri nets, is introduced, and it is shown that, for each initial marking, the reachability set of a normal marked Petri net is an effectively computable semilinear set. More generally, we show that the reachability set of a marked Petri net is an effectively computable semilinear set unless the total number of tokens in a minimal circuit is decreased to 0. We also show that a Petri net is normal if and only if it is weakly persistent for each initial marking without token-free circuits.

1. Introduction

Petri nets have been studied as models of systems of asynchronous concurrent computations. In the study of Petri nets, the reachability set (i.e., the set of markings reachable from the initial marking) has been brought to the attention as a characteristic of each system.

The reachability set of a marked Petri net may be very complicated. Hack [5] has shown that the equivalence problem for the reachability sets of arbitrary marked Petri nets is undecidable. On the other hand, for some subclasses the reachability sets have been shown to be semilinear; for example, five-dimensional vector addition systems [6], reversible Petri nets [1], persistent nets [4, 8, 9] and weakly persistent nets [10]. For these classes, the effective procedures to find the description of the reachability set in the form of a semilinear set are given. Therefore, the equivalence problem of the reachability sets and the reachability problem are decidable for these classes.

In this paper we first study sufficient conditions for a Petri net to have a semilinear reachability set, with an approach different from the above studies. We show that the reachability set of a marked Petri net is an effectively computable semilinear set unless the total number of tokens in a minimal circuit is decreased to 0 during the firings of transition sequences.

From this result we can define a class of Petri nets, called normal Petri nets, so that for each initial marking the reachability set of a normal Petri net is an effectively

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computable semilinear set. Intuitively speaking, a Petri net is normal if the total number of tokens in a minimal circuit does not decrease by firing transitions.

We also study some properties of normal Petri nets, and give a condition for a marked normal Petri net to be reversible. Finally, we show that a Petri net is normal if and only if it is weakly persistent for each initial marking without token-free circuits.

2. Definitions

Let $\mathbb{Z}$ denote the set of integers, and $\mathbb{N}$ denote the set of nonnegative integers. Let $\mathbb{Z}^X$ ($\mathbb{N}^X$) denote the set of all functions from a set X to $\mathbb{Z}$ ($\mathbb{N}$). In this paper we mostly deal with the case where X is a finite set, say $\{x_1, x_2, \ldots x_n\}$, and we identify a function f in $\mathbb{Z}^X$ with the vector $\langle f(x_1), f(x_2), \ldots, f(x_n)\rangle$. Then for functions f and g and z in $\mathbb{Z}$, the addition $f+g$, the scalar product zf, and the partial order $f \leq g$ are defined componentwise as usual. When 0 is used as a function, it denotes the zero vector $\langle 0, \ldots, 0\rangle$. So f is in $\mathbb{N}^X$ if and only if $f \geq 0$.

A set $R \subseteq \mathbb{N}^X$ is said to be *linear*, if there are elements $f_0, f_1, \ldots, f_n$ in $\mathbb{N}^X$ such that

$$R = \{f_0 + i_1 f_1 + \cdots + i_n f_n \mid i_1, \ldots, i_n \text{ are in } \mathbb{N}\}.$$

A set R is said to be *semilinear* if R is a finite union of linear sets.

A *Presburger formula* is a first-order formula over $\mathbb{Z}$ whose only atomic formulas are of the form $x + y = z$, $z \leq y$ and $x = i$ where i is a constant and x, y, z are variables. It is well known that if $p(x_1, x_2, \ldots, x_n)$ is a Presburger formula with free variables $x_1, x_2, \ldots, x_n$, then the set

$$\{\langle i_1, i_2, \ldots, i_n\rangle \geq 0 \mid p(i_1, i_2, \ldots, i_n) \text{ is true}\}$$

is semilinear [3].

A *Petri net* N is a triple (P, T, A) where P is a finite set of *places*, T a finite set of *transitions*, and A a function from $(P \times T \cup T \times P)$ to the set of nonnegative integers $\mathbb{N}$. The value $A(a, b)$ denotes the number of *arcs* from a to b. In this paper we deal with the case where each value $A(a, b)$ is 0 or 1. That is, a Petri net N is a bipartite graph, as we do not allow any multiple arcs in the graph.

A *marking* m of N is an assignment of tokens to the places, i.e., a function in $\mathbb{N}^P$. A *marked Petri net* $M = (N, m)$ is a Petri net N with a marking m, called the *initial marking* of M. We sometimes write (P, T, A, m) for (N, M) when $N = (P, T, A)$.

A transition t of a Petri net $N = (P, T, A)$ is *fireable* in a marking m if each input place of t (a place p such that $A(p, t) = 1$) has at least one token in m. In this case, the transition t may *fire* by removing one token from each input place of t and depositing one token into each output place of t (a place p such that $A(t, p) = 1$). More precisely, a transition t of a Petri net N is fireable in m if $m(p) \geq A(p, t)$ for each place p. If t fires, the resulting marking is the marking $(m + \bar{t})$, where $\bar{t}$ ($\in \mathbb{Z}^P$)

is defined by

$$\bar{t}(p) = A(t, p) - A(p, t) \quad \text{for each } p \text{ in } P.$$

We extend the notion of firing to finite sequences of transitions. The empty sequence e is fireable in every marking m, and $\bar{e} = 0$. For each t in T and w in T^*, $\overline{tw} = \bar{t} + \bar{w}$, and the sequence tw is fireable in m if t is fireable in m and w is fireable in $(m + \bar{t})$. Then the resulting marking by firing a sequence w in a marking m is $(m + \bar{w})$.

For a marked Petri net $M = (N, m)$, the reachability set $R(M)$ is the set of markings reachable from the initial marking m, i.e., $R(M) = \{m + \bar{w} \mid w \text{ in } T^* \text{ is fireable in } m\}$. It is also written as $R(N, m)$.

Assume a transition sequence w fires in a marking m. Note that the change $\bar{w}$ in the marking caused by firing w is independent of the original marking m. Moreover, while the order of transitions in w affects the fireability of w in m, the change $\bar{w}$ itself depends on the number of occurrences in w of each transition. These observations lead us to the following notions.

The *Parikh mapping* Ψ: $T^* \to \mathbb{N}^T$ is defined by

$$\Psi(w)(t) = (\text{the number of occurrences of } t \text{ in } w).$$

If $\Psi(w) \geq \Psi(w')$, then we say that w *covers* w' and write $w \geq w'$. When $\Psi(w) = \Psi(w')$, we say that w is a *rearrangement* of w', and write $w \cong w'$.

Let $N = (P, T, A)$ be a Petri net. We define the function $A: \mathbb{N}^T \to \mathbb{Z}^P$, by $A(f) = \sum_{t \in T} f(t)\bar{t}$. Clearly, A is a linear function. Since $\bar{w} = A(\Psi(w))$, if a marking m' is reachable from a marking m, then $m' = m + A(f)$ for some f in $\mathbb{N}^T$.

Since Petri nets are bipartite graphs, we can use graph-theoretical terms for Petri nets. A *path* c of a Petri net $N = (P, T, A)$ is a word $a_1 a_2 \ldots a_n$ in $(P \cup T)^*$ such that $A(a_i, a_{i+1}) = 1$ for all i, $1 \leq i < n$. A path $a_1 a_2 \ldots a_n$ is called a *circuit* if $a_n = a_1$. For a circuit c and a marking m, the *token count* $m(c)$ of c in m is the total number of tokens in the places in the circuit c, i.e., $m(c) = \sum_{p \text{ in } c} m(p)$. A circuit c is said to be *token-free* in m if $m(c) = 0$.

3. Normal Petri nets

In this section we study sufficient conditions for a marked Petri net to have a semilinear reachability set, and, then, give the definition of a normal Petri net.

Let $M = (P, T, A, m)$ be a marked Petri net and w be a sequence of transitions. Assume $m + \bar{w} \geq 0$ and no transition contained in w is fireable in m. Then we can show that there exists a token-free circuit $p_1 t_1 p_2 t_2 \ldots p_n t_n p_1$, such that $t_1 t_2 \ldots t_n$ is covered by w. Hence, if a Petri net N has no token-free circuit in every marking reachable from m, then a rearrangement of w is fireable in m if and only if $(m + \bar{w}) \geq 0$.

Lemma 3.1. *Let $M = (P, T, A, m)$ be a marked Petri net. Let w be a transition sequence* *such that $m+\bar{w}\geqslant 0$ and no transition occurring in w is fireable in m. Then there exists a token-free circuit $c=p_1t_1\ldots p_nt_np_1$ in m, such that $t_1\ldots t_n$ is covered by w.*

Proof. Let t be a transition occurring in w. Since t is not fireable in m, there exists a place p such that p is token-free (i.e., $m(p)=0$) and is an input place of t ($A(p,t)=1$). Since

$$(m+\bar{w})(p)=\bar{w}(p)=\sum_{t\in T}\Psi(w)(t)(A(t,p)-A(p,t))\geqslant 0,$$

$A(t',p)=1$ for some t' occurring in w. Since t' is not fireable in m, t' has a token-free input place p', and so on. Consequently we can get a token-free circuit $c=t_1p_nt_n\ldots t_2p_1t_1$ such that $t_1\ldots t_n\leqslant w$. □

Lemma 3.2. *Let $M=(P,T,A,m)$ be a marked Petri net. For each w in T^*, a rearrangement of w is fireable in m if:*

(1) *$m+\bar{w}\geqslant 0$, and*

(2) *for each sequence u fireable in m and each circuit $c=p_1t_1\ldots p_nt_np_1$, if $ut_1\ldots t_n\leqslant w$, then the token count of c is positive in the marking $(m+\bar{u})$.*

Proof. We prove the lemma by induction on the length of w. If $w=e$, it is clear. If $w\neq e$, then, from Lemma 3.1, there exists a transition t which occurs in w and is fireable in m. Let $w=utu'$. Then (1) $(m+\bar{t})+\overline{uu'}\geqslant 0$, and $(m+\bar{t})$ and uu' satisfy condition (2). By the induction hypothesis a rearrangement v of uu' is fireable in $(m+\bar{t})$. Thus tv is a rearrangement of w and is fireable in m. □

Theorem 3.3. *If a marked Petri net $M=(P,T,A,m)$ has no token-free circuits in every reachable marking, then the reachability set $R(M)$ is semilinear and*

$$R(M)=\{m' \mid m'=m+A(f)\geqslant 0 \text{ for some } f \text{ in } \mathbb{N}^T\}.$$

Proof. By Lemma 3.2, m' is reachable from m if and only if $m'=m+\bar{w}$ for some w in T^*, or equivalently, $m'=m+A(f)$ for some f in $\mathbb{N}^T$. Since A is a linear function, $\exists f\; m'=m+A(f)$ is a Presburger formula and $R(M)$ is a semilinear set. □

Example 3.4. Let $M=(\{1,2,3\},\{s,t,u\},A,\langle 0,0,1\rangle)$ be a marked Petri net as shown in Fig. 1. Since the token count of each circuit is positive in all reachable markings,

$$R(M)=\{m' \mid m'=m+i\bar{s}+j\bar{t}+k\bar{u}\geqslant 0 \text{ for some } i,j,k\geqslant 0\}$$

where

$$m=\langle 0,0,1\rangle,\quad \bar{s}=\langle 1,1,-1\rangle,\quad \bar{t}=\langle 0,-1,1\rangle \quad\text{and}\quad \bar{u}=\langle -1,-1,1\rangle.$$

Hence,

$$R(M)=\{m' \mid m'=\langle i-k,\, i-j-k,\, 1-i+j+k\rangle\geqslant 0 \text{ for some } i,j,k\geqslant 0\}.$$

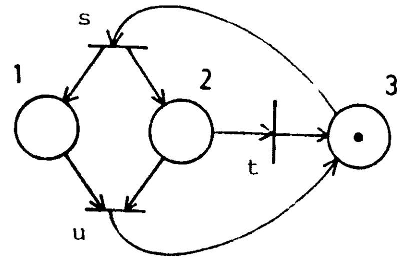

Fig. 1.

Now, we show that the reachability set is semilinear unless the token count of a minimal circuit in the net decreases to 0. A circuit c of a Petri net N is said to be ***minimal*** if the set of places in c does not properly include the set of places in any other circuit. A circuit c of a marked Petri net $M=(N, m)$ is said to be *sinkless*, if the token count of c is not decreased to 0 by firing transitions, i.e., if, for a marking m' reachable from m, and m'' reachable from m', $m'(c)>0$ implies $m''(c)>0$. A marked Petri net M is *sinkless* if each minimal circuit of m is sinkless.

For each marking m, let $F(m)$ denote the set of token-free minimal circuits in m. Then, a marked Petri net $M=(N, m)$ is sinkless if $F(m+\overline{ww'}) \subseteq F(m+\bar{w})$ for each sequence ww' fireable in m.

Theorem 3.5. *If a marked Petri net $M=(P, T, A, m_0)$ is sinkless, then we can effectively compute the reachability set of M in the form of a semilinear set.*

Proof. Let m be a reachable marking of M and H be the set of markings that are reachable from m without firing the transitions in circuits in $F(m)$. Note that to fire a transition in a minimal circuit of M, one must make the token count of the circuit positive, and once one does it the token count of the circuit is positive in all subsequent markings. Let m' be a marking reachable from m. If $F(m')=F(m)$, then m' is in H. If $F(m') \subsetneq F(m)$, then m' is reachable from m via m'' such that m'' is in H and $F(m'') \subsetneq F(m)$.

Hence, for each subset C of $F(m_0)$, we can inductively construct a formula $H_C(m, m')$ that means: m' is reachable from m, provided that m is a reachable marking of M and $F(m)=C$, as follows:

$$H_\phi(m, m') = \Gamma_\phi(m, m')$$

and

$$H_C(m, m') = \Gamma_C(m, m') \vee \bigvee_{C' \subsetneq C} (\exists m'' \Gamma_C(m, m'') \wedge \Delta_{C'}(m'') \wedge H_{C'}(m'', m'))$$

where $\Delta_C(m)$ is a formula which means that $F(m)=C$, and $\Gamma_C(m, m')$ is a formula which means that m' is reachable from m without firing the transitions in the circuits in C, provided that m is a reachable marking of M and $F(m)=C$.

To show that the formula $\Pi_{F(m_0)}(m_0, m')$ is expressible in Presburger arithmetic, it is sufficient to show that, for each C, $\Gamma_C(m, m')$ and $\Delta_C(m)$ are expressible in Presburger arithmetic. Since $F(m) = C$, $\Gamma_C(m, m')$ is true if and only if $m' = m + A(f) \geq 0$ for some f in $\mathbb{N}^T$ such that $f(t) = 0$ for each transition t in each circuit in C, from Lemma 3.2. Since A is a linear function, $\Gamma_C(m, m')$ is expressible in Presburger arithmetic. The formula $\Delta_C(m)$ is also expressible in Presburger arithmetic as follows:

$$\Delta_C(m) = \bigwedge_{c \in C} \left(\bigwedge_{p \text{ in } c} m(p) = 0 \right) \wedge \bigwedge_{c \notin C} \left(\bigvee_{p \text{ in } c} m(p) > 0 \right).$$

Hence $\Pi_{F(m_0)}(m_0, m')$ is expressible in Presburger arithmetic, and the reachability set is semilinear. □

Now we define a normal Petri net. A Petri net N is *normal* if, for each minimal circuit c and each transition t in N, $\sum_{p \text{ in } c} \bar{t}(p) \geq 0$. Hence, in a normal Petri net, if a transition t has an input place in a minimal circuit c, then t has an output place in c. Note that a conflict-free Petri net is normal [7].

Theorem 3.6. *Let N be a Petri net. The following four conditions are equivalent:*

(1) *N is normal.*

(2) *$M = (N, m)$ is sinkless for each initial marking m.*

(3) *If a marking m has no token-free circuits, then there do not exist token-free circuits in every marking reachable from m.*

(4) *If a transition t has an input place in a minimal circuit c, then t has an output place in c.*

Proof. It is easy to show that (1) implies (2), and (2) implies (3). Let $N = (P, T, A)$. To show that (3) implies (4), assume that there exist a minimal circuit $c = p_1 t_1 \ldots p_n t_n p_1$ and a transition t in N such that $A(p_1, t) = 1$ and $A(t, p_i) = 0$ for any i. Define a marking m as follows:

$$m(p) = \begin{cases} 0 & \text{if } p \text{ occurs in } c \text{ and } A(p, t) = 0, \\ 1 & \text{otherwise.} \end{cases}$$

Since c is a minimal, the token count of each circuit is positive in m. Clearly t is fireable in m, and the firing of t decreases the token count of c to 0.

Finally, we show that (4) implies (1). Let $A(p, t) = 1$ for some place p in a minimal circuit c. Then $A(t, p') = 1$ for some p' in c. Since c is minimal, $A(p'', t) = 0$ for any place $p'' \neq p$ in c. Hence, $\sum_{p \text{ in } c} \bar{t}(p) \geq 0$. □

Theorem 3.7. *If a Petri net $N = (P, T, A)$ is normal, then, for each initial marking m, we can effectively compute the reachability set $R(N, m)$ in the form of a semilinear set. Moreover, if the initial marking m has no token-free circuits, then the reachability set* *is represented as follows*:

$$R(N, m) = \{m' \mid m' = m + A(f) \geq 0 \text{ for some } f \text{ in } \mathbb{N}^T\}.$$

Proof. The proof clearly follows from Theorems 3.3, 3.5 and 3.6. □

Example 3.8. Let $N = (\{1, 2\}, \{s, t, u\}, A)$ be a Petri net as shown in Fig. 2. N has one minimal circuit $c = 2u2$, and N is normal. The formulas $\Pi_C(m, m')$ and $\Gamma_C(m, m')$ are represented as follows:

$$\Pi_\phi(m, m') = \Gamma_\phi(m, m') = \exists i \geq 0\ \exists j \geq 0\ \exists k \geq 0\ m' = m + i\bar{s} + j\bar{t} + k\bar{u} \geq 0,$$

$$\Gamma_{\{c\}}(m, m') = \exists i \geq 0\ \exists j \geq 0\ m' = m + i\bar{s} + j\bar{t} \geq 0$$

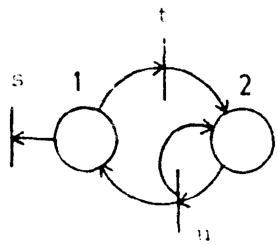

Fig. 2.

and

$$\Pi_{\{c\}}(m, m') = \Gamma_{\{c\}}(m, m') \vee (\exists m''\ \Gamma_{\{c\}}(m, m'') \wedge m''(2) > 0 \wedge \Pi_\phi(m'', m')),$$

where

$$\bar{s} = \langle -1, 0\rangle, \quad \bar{t} = \langle -1, 1\rangle \quad \text{and} \quad \bar{u} = \langle 1, 0\rangle.$$

Next, we show that, for a certain subclass of normal marked Petri nets, a marked Petri net $M = (N, m)$ is *reversible*, i.e., if m' is reachable from m, then m is reachable from m'.

Theorem 3.9. *Let $M = (P, T, A, m)$ be a normal marked Petri net. If each place of M has as many input arcs as its output arcs, and m is a marking without token-free circuits, then M is reversible.*

Proof. If m' is reachable from m, then $m' = m + A(f)$ for some f in $\mathbb{N}^T$. Let $g = ih - f$, where h is a constant function $\langle 1, 1, \ldots, 1\rangle$ and i is a sufficiently large integer so that $g \geq 0$. Since each place has as many input arcs as its output arcs, we have that $A(h) = 0$. Since $A(g) = iA(h) - A(f) = -A(f)$, we have that $m = m' + A(g)$. Thus m' is reachable from m by Theorems 3.6 and 3.7. □

A *marked directed graph* is a marked Petri net in which each place has exactly one input arc and one output arc [2]. In a marked directed graph, the token count of every circuit does not change by firing, and a marking is live if and only if the token count of each circuit is positive. Thus we can extend [2, Theorem 12], which states that a strongly connected marked directed graph with a live initial marking is reversible.

Theorem 3.10. *A marked directed graph with a live initial marking is reversible.*

4. Weakly persistent Petri nets and normal Petri nets

In this section we show that a Petri net is normal if and only if it is weakly persistent for each initial marking without token-free circuits. A marked Petri net $M=(N, m)$ is said to be *weakly persistent* if, whenever w and w' are fireable in m and w covers w', there exists a rearrangement $w'w''$ of w which is fireable in m.

In [10] it has been shown that the reachability set of a weakly persistent net is an effectively computable semilinear set, and that it is decidable whether a given marked Petri net is weakly persistent or not.

Theorem 4.1. *A Petri net N is normal if and only if the marked Petri net $M=(N, m)$ is weakly persistent for each marking m without token-free circuits.*

Proof. '*Only if*': Let w and w' be fireable in a marking m without token-free circuit and $w' \leqslant w$. Since $(m+\overline{w'})$ has no token-free circuits, there exists a w'' such that w'' is fireable in $(m+\overline{w'})$ and $w'w'' \cong w$.

'*If*': Assume a Petri net $N=(P, T, A)$ is not normal. Then there exist a minimal circuit $c=p_1t_1 \ldots p_nt_np_1$ and a transition t such that $A(p_1, t)=1$ and $A(t, p_i)=0$ for any i. Let k be a sufficiently large integer. Define a marking m as follows:

$$m(p)=\begin{cases}1 & \text{if } p \text{ occurs in } c \text{ and } A(p, t)=1,\\ 0 & \text{if } p \text{ occurs in } c \text{ and } A(p, t)=0,\\ k & \text{if } p \text{ does not occur in } c.\end{cases}$$

Since c is minimal, m has no token-free circuits. Clearly t is fireable in m and none of $t_1, \ldots, t_n$ is fireable in $(m+\bar{t})$. Since c is minimal, $A(p_i, t_j)=0$ for all $i, j, i \neq j$. Therefore the sequence $t_1 \ldots t_nt$ is fireable in m for a sufficiently large integer k. Hence the marked Petri net $M=(N, m)$ is not weakly persistent. $\square$

There exists a marked Petri net $M=(N, m)$, such that M is not weakly persistent, while the net N is normal. For example, the Petri net N shown in Fig. 2 is a normal Petri net. While s and tus are fireable in $m=\langle 1, 0\rangle$, neither t nor u is fireable in $(m+\bar{s})=\langle 0, 0\rangle$. Thus $M=(N, m)$ is not weakly persistent. It is also easily shown that the class of weakly persistent nets and the class of sinkless nets are incomparable.

Finally, we note a necessary condition for a Petri net to be weakly persistent for all initial markings. We can get a necessary condition which is slightly weaker than normality. In a normal Petri net, if a circuit $c = p_1 t_1 \ldots p_n t_n p_1$ has a transition t such that $A(p_1, t) = 1$ and $A(t, p_i) = 0$ for all i, $1 \leq i \leq n$, then c is not minimal. That c is not minimal means that there exists a path $p_i t p_j$ for some i, j, $i+1 \neq j$. But in the proof of the 'if part' of Theorem 4.1 we have shown that $M = (N, m)$ is not weakly persistent for some m, if there exist a circuit $c = p_1 t_1 \ldots p_n t_n p_1$ and a transition t such that $A(p_1, t) = 1$, $A(t, p_i) = 0$ for all i, and $A(p_i, t_j) = 0$ for all i, j, $i \neq j$. Hence we have the following theorem.

Theorem 4.2. *Let $N = (P, T, A)$ be a Petri net such that for each initial marking m, $M = (N, m)$ is weakly persistent. For each circuit $c = p_1 t_1 \ldots p_n t_n p_1$ in N, if there exists a transition t such that $A(p_1, t) = 1$, and $A(t, p_i) = 0$ for all $i, 1 \leq i \leq n$, then $A(p_i, t_j) = 1$ for some i, j, $i \neq j$.*

Remark 4.3. In this paper we place the restriction on Petri nets that there do not exist multiple arcs between places and transitions. But it is not essential to prohibit the multiple arcs from transitions to places. Allowing them we can get similar results (of course except Theorem 3.10) with minor modifications in the proofs. On the other hand, if we allow the multiple arcs from places to transitions, we may need more involved arguments.

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